Study of Numerical Method for Magnetic Shielding Problem Considering Actual Measurement of Magnetic Environment

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Abstract — For an optimal design of magnetically shielded rooms, one of the most important issues is to appropriately consider how external magnetic field, which should be shielded, are imposed in a target area. In this paper, we examined the possibility of applying the Bs method to the shielding problem with measurement-based magnetic field, which includes an irrotational field component as noise, and therefore is inconsistent with Maxwell's equations. We show that the Bs method has a well-defined physical meaning for the irrotational field component. Furthermore, we explain why the convergence and accuracy of the Bs method is better than those of the A method.

I. Introduction

In the design of magnetically shielded rooms, external magnetic fields which are imposed in the shielded area should be taken into account. To accurately understand various characteristics of the magnetic field in and around shielding rooms, the measurements are usually executed before the design of shielding rooms, because the sources of the magnetic field are unknown in most of actual cases. However, a measured data of magnetic environment contains a noise, and the estimated magnetic field (magnetic flux density) generally includes an irrotational field component, which is inconsistent with Maxwell equations.

In this paper, we examined the possibility of applying the Bs method [1], in which a given magnetic field can be treated as a noise, to such a problem having magnetic field noise. The inconsistent field component is converted to the distribution of the equivalent magnetizing current by the proposed method. The convergence characteristics of the ICCG method, when the edge-based finite element method is used, are examined in an example having a random noise.

II. Physical meaning of Bs method for Irrotational Field Component

The magnetic field **B**s(**r**,t), which can be obtained from the measurement at grid points, is interpolated by some numerical techniques. It is inconsistent with Maxwell's equations because of the various error caused by the measurement noise and the interpolation, In this case, the divergence-free condition (Gauss's law for magnetism) and its invariable condition (time derivative of Faraday's law) are not satisfied. However, if an equivalent current density is calculated for the measured flux density **B**s(**r**,t), the governing equations using the magnetic vector potential **A**(**r**,t), which corresponds to the measured magnetic field **B**s(**r**,t), can be solved. In this case, the calculated flux density **B**(**r**,t) satisfies the divergence-free condition and the magnetic field **B**(**r**,t) differs from the original field **B**s(**r**,t). In other words, original field **B**s(**r**,t) can be uniquely decomposed into the sum of an irrotational field component and a divergence-free field component **B**(**r**,t) according to the Helmholtz decomposition theorem [2], so that Bs method eliminates the influence of the irrotational field component from the solution of the governing equations. Furthermore, as the equivalent current density is continuous, the Bs method is suitable for the ICCG method [3]. Therefore, the Bs method has a well-defined physical meaning for the irrotational field component contained in measurement-based data.

III. Test Calculation and Discussion

The Bs method is applied to the edge-based finite element method and is solved using the ICCG method with an acceleration factor. Figure 1 shows the magnetic shielding box model. The shielding box is composed of six magnetic material plates whose thickness is equal to 1 mm. The relative permeability and conductivity are 10000 and 1.67E+6 S/m, respectively. The external magnetic field which is imposed on the box is perpendicular to the plate into the y-z plane (Case A) or oblique (Case B) as shown in Fig.1.

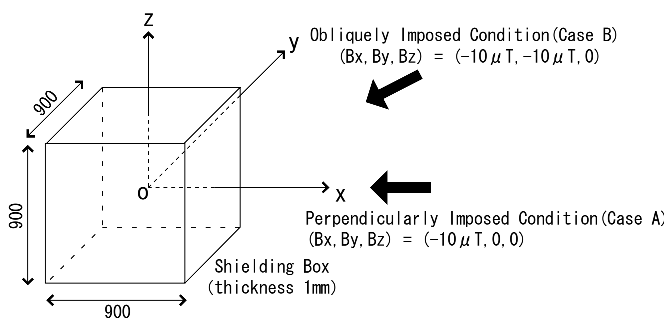

Fig. 1. Magnetic shielding box model and imposed conditions of external magnetic field.

Figure 2 shows the convergence process of Bs method. The convergence of Bs method (open circle) is faster than that of A method (open triangle). Figure 3 shows the calculation results of magnetic field component Bx on y-axis in the box. The results of Bs method (triangle) and A method (circle) are shown. The relative error of the ICCG method are 1.0E-4 (filled mark) and 1.0E-8 (open mark), respectively. It can be understood that the convergence and accuracy of the Bs method is better than those of the A method due to the following reason:

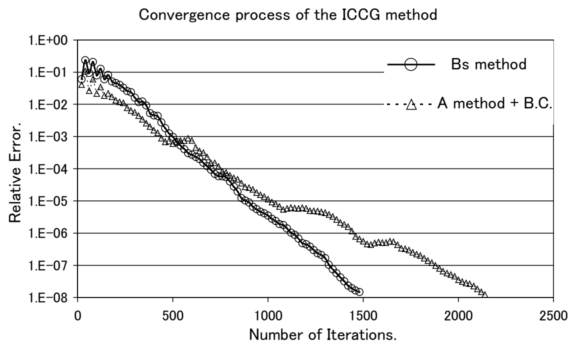

Fig. 2. Convergence process of ICCG method for Bs method and A method (Case A).

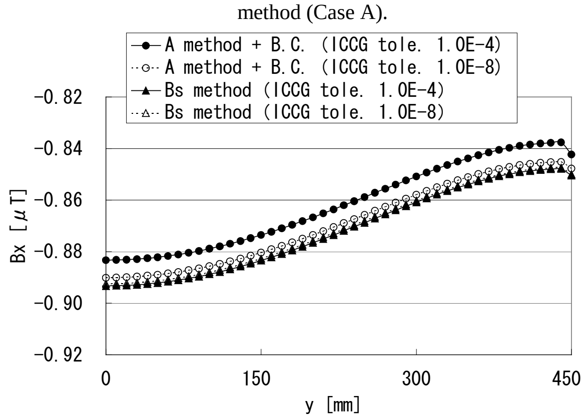

Fig. 3. Calculation result of Bx on y-axis.

The ICCG method is based on the Krylov subspace $span\{\mathbf{r}_0,\ \mathbf{K}\mathbf{r}_0,\ \mathbf{K}^2\mathbf{r}_0,\ ....\}$ (where, $\mathbf{r}_0$: an initial residual vector ($\mathbf{r}_0 = \mathbf{b} - \mathbf{K}\mathbf{x}_0$), $\mathbf{x}_0$: an initial solution vector $\mathbf{x}_0$.) [4] for a matrix equation $\mathbf{K}\mathbf{x} = \mathbf{b}$, where approximate solutions are searched step by step. In comparison between Bs method and A method, the principal difference is the residual vectors $\mathbf{r}_0$, whereas the coefficient matrixes $\mathbf{K}$ of both methods are almost the same although the convergence of the ICCG method is explained by the condition number of the coefficient matrix [5]. In the Bs method, non-zero elements of the initial residual vector are dispersed in whole elements where the equivalent current density, which corresponds to magnetic field **B**s(**r**,t), is imposed. On the other hand, in the A method, the non-zero elements are restricted within the positions which correspond to edge-elements just located on the boundary surface. In other words, the Krylov subspace of the A method is gradually extended from the boundary surface to the inner region through the structure of the coefficient matrix **K**.

Figure 4 shows the effect of measurement error η, which corresponds to the ratio (percentage) of the random noise to 10 µT noise, on the number of iterations of the ICCG method and CPU time for Case B. The random noise is generated by the uniform distribution. The magnetic field **B**s(**r**,t) contains a certain irrotational field component because the random noise is added independently in each mesh. However, the figure confirms that the irrotational component has no influence to the convergence characteristic, because the irrotational field component is eliminated in the Bs method as mentioned above.

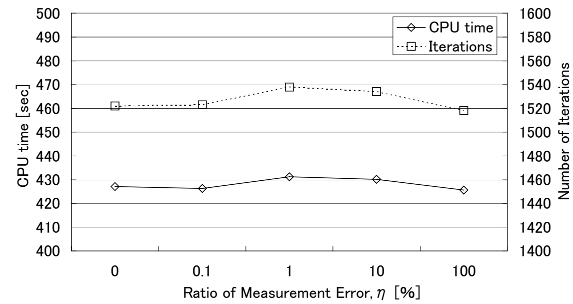

Fig. 4. Number of iterations of ICCG method and CPU time for each measurement error (Case B).

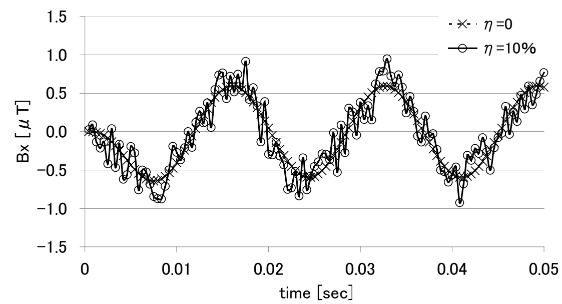

Fig. 5. Waveform of shielded magnetic field at origin point in the box (60Hz, CaseB).

Figure 5 shows the waveforms of magnetic field component Bx at the origin point in the box when an ac magnetic field (60 Hz) is applied. It is concluded that the Bs method can be used even in the eddy current problem although **B**s(**r**,t) is inconsistent with the Faraday's law.

IV. Conclusion

We conclude that the Bs method has a well-defined physical meaning, because any irrotational field component which is included in measurement-based flux densities is eliminated. Furthermore, the Bs method is suitable for the ICCG method, and then the convergence and accuracy of the Bs method is better than those of the A method.

V. References

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